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WATER WAVE TRANSMISSION THROUGH AND REFLECTION BY PERVIOUS COASTAL STRUCTURES
Hydraulic Laboratory Investigation
by
A. M. Kamel

October 1969

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CORPS OF ENGINEERS
Vicksburg, Mississippi

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FOREWORD

Authority for the U. S. Army Engineer Waterways Experiment Station to conduct the study reported herein, designated Engineering Study No. 853, "Investigation of Wave Reflecting and Transmitting Characteristics of Rubble-Mound Breakwaters, Rubble Wave Absorbers, Sand Beaches, Wave Traps, and Resonators," was contained in a letter from the Office, Chief of Engineers, dated 30 August 1957.

The investigation was conducted during the period December 1965 to September 1966 in the Hydraulics Division of the Waterways Experiment Station under the general direction of Mr. E. P. Fortson, Jr., Chief of the Hydraulics Division, and Mr. R. Y. Hudson, Chief of the Wave Dynamics Branch. The experiments were conducted by Messrs. P. K. Senter and D. D. Davidson, Engineers, Wave Dynamics Branch, under the supervision of Dr. A. M. Kamel, Special Assistant for Research to the Chief, Hydraulics Division. This report was prepared by Dr. Kamel.

Liaison with the Office, Chief of Engineers, was maintained throughout the course of the investigation by means of progress reports and conferences. Mr. C. E. Lee, Assistant Chief, Hydraulic Design Branch, Engineering Division, Civil Works, Office, Chief of Engineers, visited the Waterways Experiment Station at various times in connection with the study.

Directors of the Waterways Experiment Station during the conduct of this study and the preparation of this report were COL John R. Oswalt, Jr., CE, and COL Levi A. Brown, CE. Technical Directors were Mr. J. B. Tiffany and Mr. F. R. Brown.
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NOTATION

\( a \) Constant of the fluid and porous medium; also half the wave height
\( a_n \) Wave amplitude, \( n = 0, 1, 2, 3 \)
\( a_0 \) An alternate symbol for amplitude of wave entering a porous medium
\( a_1 \) Amplitude of wave transmitted through the porous medium
\( a_2 \) Amplitude of wave reflected by the porous medium
\( a_3 \) Amplitude of wave reflected by absorber beach
\( c \) Constant depending on the porosity of the medium
\( d \) Grain diameter
\( E_l \) Energy loss per unit volume of porous structure during period \( T \)
\( f \) Functions; also the Fanning friction factor, equation 9
\( F_x \) Component body force per unit volume acting on the fluid in the \( x \) direction
\( g \) Gravity acceleration
\( h \) Displacement of water surface
\( H \) Water depth
\( H_1, H_2 \) Water level upstream and downstream of porous structure, respectively
\( k \) Coefficient of resistance of porous structure
\( k_x \) Permeability of porous medium in \( x \) direction
\( K \) Constant in a general formula for damping, equation 33
\( K_r \) Wave reflection coefficient; also \( a_2/a_0 \)
\( K_t \) Wave transmission coefficient; also \( a_1/a_0 \)
\( L \) Length of porous structure
\( m, n \) Exponents
\( N \) Constant in the formula of damping, equation 28
\( N_R \) Reynolds number
\[ \frac{\partial p}{\partial x} \] Pressure drop with distance

\[ Q \] Flow discharge

\[ swl \] Still-water level

\[ t \] Time

\[ T \] Wave period

\[ u \] Macroscopic velocity in \( x \) direction

\[ u_s \] Maximum horizontal particle velocity at the water surface

\[ u_1, u_2 \] Fluid velocity upstream and downstream of porous structure, respectively

\[ W \] Flux of energy during period \( T \) at a section

\[ x \] Horizontal distance along direction of wave motion

\[ z \] Elevation from a given horizontal datum (e.g., channel bottom)

\[ \alpha \] Shape factor; also damping factor

\[ \beta \] Constant of proportionality, equation 29

\[ \gamma \] Constant of proportionality, equation 50

\[ \Gamma \] Constant, equation 32

\[ \Delta E \] Energy loss per unit volume of porous structure per unit time

\[ \Delta H \] Difference in levels of water surface on the two sides of a porous structure

\[ \varepsilon \] Porosity (void ratio); also phase angle

\[ \eta_x \] Displacement of envelopes of wave amplitude from still-water level

\[ \lambda \] Wavelength

\[ \mu \] Absolute viscosity of liquid

\[ \nu \] Kinematic viscosity of liquid

\[ \pi \] Numerical constant

\[ \rho \] Mass density of fluid

\[ \sigma \] Angular frequency

\[ \omega \] Wave velocity

\[ \Omega \] Potential energy per unit mass
British units of measurement used in this report can be converted to metric units as follows:

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<th>To Obtain</th>
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</thead>
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<tr>
<td>inches</td>
<td>2.54</td>
<td>centimeters</td>
</tr>
<tr>
<td>feet</td>
<td>0.3048</td>
<td>meters</td>
</tr>
<tr>
<td>feet per second</td>
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SUMMARY

An expression is given for the resistance of free surface flow in a porous medium, such as a rubble-mound breakwater, in terms of the density of the fluid, the approach velocity, the grain diameter of the material in the breakwater structure, and a resistance coefficient which is a function of Reynolds number of the flow. Tests indicated that the length of the structure relative to the grain diameter is an important factor in determining the resistance of free surface flow in a porous medium. A certain ratio between the length of the structure and its grain diameter must be reached before the structure can be treated as a porous medium. An expression derived by Dr. G. H. Keulegan for damping of translation waves by screen filters was modified to give the transmission coefficient of surface waves propagated through a porous structure. The constants in this expression were also found to be a function of the length of the structure relative to the grain diameter. Measured and computed values of wave transmission coefficients agreed well.
WATER WAVE TRANSMISSION THROUGH AND REFLECTION BY
PERVIOUS COASTAL STRUCTURES

Hydraulic Laboratory Investigation

PART I: INTRODUCTION

Background

1. When a water wave approaches a pervious coastal structure such as a rubble-mound breakwater, part of the wave energy is dissipated on the face and through the structure, part is reflected, and the rest is transmitted through the structure. The parameters defining these phenomena are the characteristics of the incident wave (amplitude, period, and water depth) and the characteristics of the breakwater (grain size distribution of the material, voids ratio, and the general configuration and width of the structure). The structure always has a complex form, the width varying with the depth, with layers of different grain size superimposed on each other, thus defying an exact analysis.

2. One of the difficulties in model studies of water wave problems is the simulation of the wave reflection by and transmission through pervious full-scale structures such as rubble-mound breakwaters. The dimensions of the model breakwaters are selected based on Froude's law of similarity. However, viscous forces may cause scale effects in the flow through the pores of the model breakwater, and the flow in the model might be laminar whereas flow in the prototype is turbulent.

3. The general problems of wave reflection by and transmission through wave filters and wave absorbers have been the subject of several investigations, such as those by Biesel; O'Brien and Chaffin; Bowers and Herbich; Herbert; Straub et al.; Costello; Goda and Ippen; Le Méhaute; Kondo; Johnson et al.; and Keulegan.

4. A systematic approach to determine the wave transmission and reflection characteristics of porous structures would include:
   a. Permeability tests in which a segment of a test flume is filled with experimental material (different geometric shapes...
such as spheres, cubes, etc., would be used, with upstream and downstream faces of the structure vertical). During such tests, various constant discharges would be passed through the flume and the differences in the levels of the water surfaces on the two sides of the test structure would be noted. The effect of grain size, shape, roughness, and porosity on the dissipativeness of the porous structure would also be studied, the dissipativeness being a fundamental property of a porous structure in the absorption of wave energy.

b. Tests to determine the wave transmission and reflection characteristics of porous structures. In these tests the upstream and downstream faces of the structure would be vertical, and various water depths, wavelengths, wave heights, and structure lengths would be used.

c. Wave transmission and reflection tests using breakwaters with a range of face slopes common to such structures. These tests would be conducted using test sections and armor units similar to those used by the U.S. Army Corps of Engineers in the construction of rubble breakwaters and jetties.

The study reported herein was concerned with items a and b above.

### Purpose and Scope of Study

5. The purpose of Corps of Engineers Engineering Study No. 853, "Investigation of Wave Reflecting and Transmitting Characteristics of Rubble-Mound Breakwaters, Rubble Wave Absorbers, Sand Beaches, Wave Traps, and Resonators," is to determine optimum designs for rubble breakwaters, rubble wave absorbers, and other types of absorbing devices, with respect to wave reflection, absorption, and transmission. The study includes:

a. A critical review of the theoretical aspects of wave absorbers (natural sand beaches, wave traps, resonators, and rubble-mound absorbers) to determine how the findings can be adapted to actual situations; a special theoretical investigation to determine the feasibility of using rubble-mound or other porous types of construction for absorbing wave energy inside harbors; and a study of scale effects related to wave absorbers and the conduct of an experimental investigation to reduce scale effects to a minimum.

b. Tests to determine the most accurate method of determining the wave reflection and transmission coefficients of rubble-mound breakwaters and wave absorbers.
Tests of rubble-mound wave absorbers to determine the ratio of reflected to incident wave heights as a function of structure slope, porosity of cover layer, thickness of cover layer, weight and shape of armor unit, dimensions of waves, and depth of water.

d. Tests to determine the percentage of wave energy transmitted through and over rubble-mound structures to develop design criteria for the protection of inner-harbor areas.

e. Tests to determine the wave-absorbing characteristics of sand beaches, wave traps, and resonators.

f. Tests to investigate the absorbing characteristics of various types of floating breakwaters for marinas located in deep-water lakes and reservoirs.

6. The purpose of the study reported herein, which was the initial phase of Engineering Study No. 853, was to provide an analytical basis, and outline an experimental investigation, for evaluation of the scale effects related to the wave reflection and transmission characteristics of rubble-mound breakwaters and jetties.
Steady Flow in Porous Media

7. As stated by Muskat,\(^{14}\) the flow of viscous fluid through porous media is a special case of the general problem of the viscous flow of fluids between impermeable boundaries. As long as the pores of a medium are fixed and their bounding surfaces are geometrically describable, the flow through these pores is subject to detailed description by means of the classical equations of hydrodynamics (continuity, state, and dynamical). However an inspection of treatises on hydrodynamics indicated that, except for certain cases of simple geometry, the mathematical difficulties in the solution of the classical equations are insurmountable.

8. The difference between the hydrodynamics of flow through porous media and the classical theory of viscous flow lies in the expression of the dynamical equations, which the classical theory presents in the form of Navier-Stokes equations. The law of conservation of matter and the thermodynamic definition of a fluid must be retained in any hydrodynamic system. However, it is reasonable that the dynamical reactions of a fluid passing through the fine channels of a porous medium may, from a macroscopic point of view, appear in quite different form than when analyzed microscopically and represented by the Navier-Stokes equations. It is this difference that has been empirically established by the early experiments of Darcy and has been formulated as Darcy's law.

9. For a porous medium the dynamic equation of motion may be written as

\[
\varepsilon \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \varepsilon = - \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{\partial \sigma}{\partial x} - \frac{k}{d} u^2
\]

(1)

Assuming steady flow and neglecting the term \(u \frac{\partial u}{\partial x}\), equation 1 reduces to

\[
\frac{k}{d} u^2 = - \frac{1}{\rho} \frac{\partial}{\partial x} (p + \rho g z)
\]

(2)
Assuming that

\[ k = c \left( \frac{ud}{v} \right)^{-m} \]  

(3)

where

\[ c = c(\varepsilon) \]

yields

\[ \frac{c}{d} \left( \frac{ud}{v} \right)^{-m} u^2 = -\frac{1}{\rho} \frac{\partial}{\partial x} (p + \rho gz) \]  

(4)

or

\[ u^{2-m} = -\frac{1}{c} \frac{d}{v \rho} \frac{\partial}{\partial x} (p + \rho gz) \]  

(5)

Equation 5 can be expressed in the form

\[ \left( \frac{\partial p}{\partial x} - F_x \right) = -au^n \]  

(6)

When the flow is laminar, equation 6 becomes the well-known Darcy's law

\[ \left( \frac{\partial p}{\partial x} - F_x \right) = -\frac{u}{k} \]  

(7)

The range of validity of Darcy's law can be found graphically by plotting the dimensionless Fanning friction factor \( f \), used in hydraulics, against Reynolds number \( \text{Re} \)

\[ N_R = \frac{ud}{v} \]  

(8)

The factor \( f \) is defined by

\[ f = \frac{d}{2\rho u^2} \cdot \frac{\Delta p}{\Delta L} \]  

(9)

where \( \Delta p \) is the pressure difference over a length of porous medium \( \Delta L \), measured along the line of flow, and \( d \) is the grain diameter.
10. Determination of the extent of wave transmission through porous coastal structures, such as rubble-mound breakwaters, can be achieved by performing permeability, wave transmission, and wave reflection tests. In the permeability tests, a segment of the flume is filled with experimental grains (upstream and downstream faces of the porous structure are vertical), and with a given discharge \( Q \), the difference in the levels of the water surface on the two sides of the test structure is noted (fig. 1). During such tests, the effects of grain size, shape, roughness, and porosity on the dissipativeness of the structure are studied. The dissipativeness \( \Delta E \) denoting the loss of energy per unit time and per unit volume of structure is a fundamental property of porous structures in the absorption of wave energy. Dependence of \( \Delta E \) on the grain size \( d \), porosity \( \varepsilon \), and shape factor \( \alpha \) is given by

\[
\Delta E = \frac{\rho u^3}{d} f \left( \frac{ud}{v}, \varepsilon, \alpha \right) = k \frac{\rho u^3}{d} \tag{10}
\]

11. From fig. 1, the energy loss per unit volume of porous structure per unit time is

\[
\Delta E = \frac{\rho u g \Delta H}{L} \tag{11}
\]

From equations 10 and 11

\[
k = \frac{\Delta H \, g d}{L \, u^2} \tag{12}
\]
where

\[
\Delta H = \left( H_1 + \frac{u_1^2}{2g} \right) - \left( H_2 + \frac{u_2^2}{2g} \right)
\]  

(13)

Plotting \( k \) versus \( N_R \) will enable us to determine \( c \) and \( n \) for the different regions of flow in the expression

\[
k = c \left( \frac{ud}{v} \right)^{-n}
\]  

(14)

Following the permeability tests, wave transmission and reflection tests were conducted. In these tests, as in the permeability tests, a segment of the flume was filled with experimental grains, with upstream and downstream faces of the test structure vertical. The values of \( c \) and \( n \) that were determined from the permeability tests were then used in the mathematical formulations upon which the wave transmission and reflection tests were based.

**Transmission of Long Waves in Porous Media**

12. The work done during a time equal to the period of the wave can be expressed, according to Keulegan,\(^{13}\) as

\[
W = \rho \omega \int_0^T \int_0^H u^2 \, dz \, dt
\]  

(15)

This flux of energy is proportional to wave energy per wavelength, so that if \( 2a \) is the wave height,

\[
\frac{dW}{W} = 2 \frac{da}{a}
\]  

(16)

The energy loss per unit volume of porous medium per unit time is

\[
\Delta E = k \frac{\rho u^3}{d}
\]  

(17)
Since \( k = c \left( \frac{ud}{\nu} \right)^{-1} \), for small values of \( N_R \),

\[
\frac{ud}{\nu} < 1.00
\]  

(18)

Hence

\[
\Delta E = \rho \frac{cv}{d^2} u^2
\]  

(19)

The energy loss per unit volume during period \( T \) is

\[
E_L = \rho \frac{cv}{(1 - \varepsilon)} d^2 \int_0^T \int_0^H u^2 \, dz \, dt
\]  

(20)

Energy lost per unit volume of porous medium during period \( T \) must equal the space rate of decrease of energy flux; that is,

\[
\frac{dW}{dx} = -E_L
\]  

(21)

From equations 15 and 20,

\[
\frac{dW}{dx} = - \frac{cv}{(1 - \varepsilon)} \frac{W}{d^2 \, w}
\]  

(22)

Since \( W \propto a^2 \), \( \frac{dW}{W} = 2 \frac{da}{a} \), and \( w = \frac{\lambda}{T} \), equation 22 reduces to

\[
\frac{da}{a} = - \frac{1}{2} \frac{cvT}{(1 - \varepsilon)} \frac{dx}{\lambda}
\]  

(23)

The solution is

\[
a_1 = a_0 e^{-\frac{cx}{\lambda}}
\]  

(24)
where \( a_o \) is the amplitude of waves entering the porous structure and \( \alpha \) is the damping factor defined as

\[
\alpha = \frac{1}{2} \frac{cT}{d^2} \frac{2}{(1 - \varepsilon)}
\]  

(25)

If the length of the porous structure is \( L \), equation 24 becomes

\[
a_L = a_o e^{-\frac{\alpha L}{\lambda}}
\]  

(26)

Equations 25 and 26 are similar to Keulegan's equation 31 in reference 13 and give the law of energy dissipation due to porous media for waves of infinitesimal height.

For large particle velocities, the resistance coefficient of porous media can be expressed as

\[
k = c \left( \frac{ud}{v} \right)^{-n}
\]

(14 bis)

Assuming that

\[
u = \frac{\omega H}{H}
\]

(27)

the formula given by Keulegan\(^{13}\) can be modified for porous media as follows:

\[
\left( \frac{a_o}{a_1} \right)^{1-n} = 1 + \frac{1 - n}{2} Nc \frac{v}{d} \left( \frac{\varepsilon}{1 - \varepsilon} \right)^{n+1} \lambda^{1-n} T^n \left( \frac{a_o}{H} \right)^{1-n} \frac{L}{\lambda}
\]

(28)

Equation 28 can also be written as

\[
\left( \frac{a_o}{a_1} \right)^{1-n} = 1 + \beta \left( \frac{a_o}{H} \right)^{1-n} \frac{L}{\lambda}
\]

(29)

where
\[ \beta = \frac{1 - n}{2} N c \frac{\nu^n}{d^{n+1} \left( \frac{\varepsilon}{1 - \varepsilon} \right)^{1-n} T^m} \]  

(30)

where for cnoidal waves

\[ N = \int_0^\lambda \left( \frac{h}{a} \right)^2 \left( \frac{h}{a} \right)^{1-n} \, dx + \int_0^\lambda \left( \frac{h}{a} \right)^2 \, dx \]  

(31)

and for sinusoidal waves \( N \) can be expressed as

\[ N = \frac{2}{\sqrt{\pi}} \frac{\Gamma \left( 2 - \frac{n}{2} \right)}{\Gamma \left( \frac{5}{2} - \frac{n}{2} \right)} \]  

(32)

For cnoidal waves the wave profile must be known in order to obtain the value of \( N \). For sinusoidal waves, however, the following set of values is given.

<table>
<thead>
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<th>( n )</th>
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<tbody>
<tr>
<td>0.00</td>
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<tr>
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</tr>
<tr>
<td>1.00</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Small changes from sinusoidal form hardly affect the value of \( N \).

Transmission of Surface Waves Through Porous Media

14. The formulation presented in paragraphs 12 and 13 relates to transmission of translation waves such as tidal waves. A rigorous treatment of transmission of surface waves is difficult. One complication is that when wavelengths differ but little from water depths the vertical component of the particle velocity becomes appreciable. Therefore, the resultant velocity is not directed normal to the porous structure. Since no permeability tests were conducted for the condition in which the current
strikes the porous structure obliquely, we lack the necessary information from the determination of losses for finite vertical velocities. However, following the analysis by Keulegan, the expression for wave transmission through porous media is

\[
\left( \frac{a_0}{a_1} \right)^{l-n} = 1 + \frac{1 - n}{2} cK \frac{v^n}{d^{n+1} \left( \frac{\varepsilon}{1 - \varepsilon} \right)} (gh)^{l-n} \left( \frac{T}{\lambda} \right)^{l-n} \left( \frac{a_0}{H} \right)^{l-n} \frac{L}{\lambda} \tag{33}
\]

Wave Reflection and Transmission Coefficients

15. The amplitudes of the incident and reflected waves in a wave system (fig. 2) can be determined by using a single wave rod, recording the fluctuation of the water surface while moving along the channel axis for a distance not less than a quarter of a wavelength (fig. 3). The amplitude can then be obtained from the recorded wave envelopes (fig. 4) as follows. Assuming that the incident and the reflected waves are of sinusoidal form, they both must have the same period and can be expressed as

\[
\eta_0 = a_0 \sin (kx - ct - \sigma \varepsilon_0) \tag{34}
\]

\[
\eta_2 = a_2 \sin (kx + ct + \sigma \varepsilon_2) \tag{35}
\]

Assuming the phase angle for reflection \( \pi \),
Fig. 3. General view of Saginaw screw and movable wave rod

Fig. 4. Wave measurements recorded by stationary wave rod and by movable wave rod
\[ \eta = \eta_0 + \eta_2 = a_0 \sin kx \cos \omega t - \cos kx \sin \omega t + a_2 \left( \sin kx \cos \omega t + \cos kx \sin \omega t \right) \]  

(36)

Differentiating equation 36 with respect to the time \( t \) and setting the result equal to zero, the time at which this combined wave system produces maximum and minimum amplitudes is

\[ \tan \omega t_{\text{max}} = \frac{(a_2 - a_0) \cot kx}{a_0 + a_2} \]  

(37)

Substituting the values of \( \sin \omega t_{\text{max}} \) and \( \cos \omega t_{\text{max}} \) obtained from equation 37 in equation 36, the equation for the variation of wave amplitude with distance is

\[ \left| \eta_x \right| = \sqrt{(a_0 + a_2)^2 \sin^2 kx + (a_0 - a_2)^2 \cos^2 kx} \]  

(38)

Differentiating equation 38 with respect to \( x \) and setting the result equal to zero, the maximum and minimum values of \( \eta_x \) are

\[ \left| \eta_x \right|_{\text{min}} = a_0 - a_2 \quad \text{and occurs when} \quad kx = n\pi, \quad n = 0, 1, 2... \]  

(39)

\[ \left| \eta_x \right|_{\text{max}} = a_0 + a_2 \quad \text{and occurs when} \quad kx = \frac{2n + 1}{2} \pi, \quad n = 0, 1, 2... \]  

(40)

From equations 39 and 40,

\[ a_0 = \frac{1}{2} \left( \left| \eta_x \right|_{\text{max}} + \left| \eta_x \right|_{\text{min}} \right) \]  

(41)

and

\[ a_2 = \frac{1}{2} \left( \left| \eta_x \right|_{\text{max}} - \left| \eta_x \right|_{\text{min}} \right) \]  

(42)

Recording the wave envelopes upstream and downstream of the test structure,
the reflection and transmission coefficients, respectively, can be obtained as

\[ K_r = \frac{a_2}{a_0} \]  
\[ K_t = \frac{a_1}{a_0} \]  

In the present study it was found that \( a_3 \), the amplitude of the wave reflected by the absorber beach, was of negligible magnitude. Therefore, the amplitude of the transmitted wave, \( a_1 \), was recorded using a fixed wave rod located downstream of the porous structure.
PART III: EXPERIMENTAL EQUIPMENT AND PROCEDURES

Wave Channel and Wave Generator

16. Tests were conducted in a wave channel 1 ft* wide and approximately 100 ft long. The wave generator consisted of a plate hinged to the channel bottom where the depth was 2.9 ft. In front of the generator the channel bottom was inclined upward, rising for a distance of 9.0 ft. At the point where the rise terminated, the channel depth was 1.6 ft, and this depth remained constant for the remaining length of the channel (about 70 ft). For the experiment, a beach of permeable material (13 ft long) was provided which resulted in almost total absorption of energy of the incident waves.

Measuring Apparatus

17. Wave measurements were made using wave rods, amplifiers, and a multiple-channel oscillograph. Two types of wave rods were used, a fixed and a movable one. The fixed rod, a parallel wire type, was used for measuring the waves transmitted through the breakwater since reflection from the beach was negligible. The movable wave rod, attached to a Saginaw screw (fig. 3), was used for measuring the height of the incident and reflected waves in front of the structure. The wave rod traversed along the axis of the channel for a distance not less than one-quarter of a wavelength. The resulting wave trace showed superpositioning of the incident and reflected waves. Fig. 4 shows an example of the two different traces recorded by a movable and by a fixed wave rod. The envelope of the crest and trough of the wave trace recorded by the movable wave rod has a minimum height \( a_0 - a_2 \) and a maximum height \( a_0 + a_2 \) from which the amplitudes of the incident and reflected waves can be determined from equations 41.

* A table of factors for converting British units or measurement to metric units is presented on page ix.
For the permeability tests a micromanometer (Fig. 5) was used to determine the head drop from the upstream to the downstream limits of the test structures.

**Experimental Procedures**

For the permeability tests, a segment of the flume was filled with spheres or cubes, with the upstream and downstream faces of the structure vertical, a discharge $Q$ was introduced, and the difference in the levels of the surface water on the two sides of the porous medium was noted. Tests of wave transmission and reflection also involved filling a segment of the flume with spheres or cubes, with the upstream and downstream faces of the structure vertical. In these tests, water depth was held constant and wavelengths, wave heights, and lengths of the porous structures were varied. Wave transmission and reflections were observed.
Selected Test Conditions

19. In permeability tests the water depth upstream of the test section was kept constant at 1.0 ft and the wave conditions used in the tests varied over the maximum practical ranges permitted by the wave generator. The wave period $T$ varied from 0.71 to 1.94 sec, and the wave height varied from 0.006 to 0.500 ft. All waves were nonbreaking.

Type of Structures Tested

20. The structures tested were pervious vertical-face breakwaters built of spheres or cubes of different sizes. The porosity of the breakwater section was kept constant at 40 percent. The spheres used varied in diameter from $5/8$ to $2-1/2$ in., the surface being smooth. Both glass and rubber spheres were tested. The linear dimension of the cubes used varied from $1/2$ to $1-5/16$ in. The cubes were made of concrete and plastic. Cages consisting of wire screen were used in order to keep the faces of the breakwater vertical. The cages were calibrated to determine their effect on the permeability of the structure and on the wave reflection and transmission.
PART IV: ANALYSIS OF TEST RESULTS

Permeability Tests

21. In these tests the porosity of the structures tested was kept constant at 40 percent since it was found experimentally that assemblages of spheres, or even sand particles, have porosities averaging about 40 percent in spite of careful efforts to induce closer packing, and even though the predominant array in the assemblage is rhombohedral with a porosity of only 26 percent.\(^{14}\)

22. Permeability test results are plotted in plates 1 and 2 as the relation between the resistance coefficient \(k\) plotted against \(N_R\) in logarithmic coordinates when \(k = \frac{\Delta H}{L} \frac{gd}{u^2}\) and \(N_R = \frac{ud}{v}\). In the present tests the value of \(N_R\) was always larger than 1.0. However, results of tests of Bakhmeteff and Feodoroff\(^{16}\) for \(N_R < 1.0\) are included in plate 1 to define the relation between \(k\) and \(N_R\) for small values of \(N_R\). The scatter in the test results shown in plate 1 for larger values of \(N_R\) is due to the difference in grain shape and roughness.

23. In the region of small Reynolds number the data are fitted by a straight line:

\[
k = c\left(\frac{ud}{v}\right)^{-1}, \quad c = 630, \quad \frac{ud}{v} < 10
\]  

(45)

In the region of moderately large Reynolds number the data are better defined by a curve. For simplicity, however, this can be approximated by a straight line:

\[
k = c\left(\frac{ud}{v}\right)^{-1/2}, \quad c = 170, \quad 10 < \frac{ud}{v} < 100
\]  

(46)

In the region of high Reynolds number the data are approximated by a straight line:

\[
k = c\left(\frac{ud}{v}\right)^{-0.23}, \quad c = 62, \quad \frac{ud}{v} > 100
\]  

(47)
for spheres (see plate 2a) and

$$k = c \left( \frac{ud}{v} \right)^{-0.21} , \quad c = 50 , \quad \frac{ud}{v} > 100$$  \hspace{1cm} (48)

for cubes (see plate 2b). The different values for c and n for $N_R > 100$ when plotted against $L/d$, the ratio between the length of the structure and the grain diameter, showed that c and n increase for increasing values of $L/d$ (see plate 3 and table 1).

Wave Transmission Tests

Transmission of long waves in porous media

24. Equations 26 and 29 are two expressions for damping for waves of infinitesimal height and waves of finite height, respectively. Since infinitesimal waves cannot be accurately handled, the expected reductions of height for the condition must be inferred by projections from observations applying to waves of finite wave height as in plates 4 through 9. The projection of waves in the region of small amplitudes is not certain. Corresponding to different absorber lengths $L/\lambda$, the reduction ratios $(a_1/a_0)$ for the infinitesimal waves by adapted projections were determined. These values together with the reduction ratios for other wave heights as read from the curves in plates 4 through 9 are listed in table 2 and are plotted with semilogarithmic coordinates against the relative lengths of the absorbers in plates 10 through 14. Those of the infinitesimal waves are shown as closed circles. When the points are fitted by a straight line in accordance with equation 26, the logarithmic decrement $\alpha$ can be evaluated from the slope of the line. The $\alpha$ values are given in table 3.

25. From the presumed values of $\alpha$, and using equation 25, the corresponding values of c were calculated and are given in table 3. The agreement between the calculated values of c given in table 3 and the values obtained from the permeability tests, $c = 630$, is poor.

26. The inferiorly placed curves in plates 10 through 14 relate to waves of finite amplitude. Divergence from the upper limiting curve increases with increasing magnitude of the initial wave height $a_0$. Thus,
the validity of equation 25 is restricted to very small waves. As regards the waves of finite amplitude, equations 29 and 30 are used. That is,

\[
\left( \frac{a_0}{a_1} \right)^{1-n} = 1 + \beta \left( \frac{a_0}{H} \right)^{1-n} \frac{L}{\lambda}
\]

where

\[
\beta = \frac{cN}{2} \frac{\nu^n}{d^{n+1}} \left( \frac{\varepsilon}{1 - \varepsilon} \right)^{\lambda-1} \left( \frac{a_0}{H} \right)^{1-n} T^n
\]

The minimum value of \( \bar{u}_S \), is 0.48 and 0.515 fps for cubes and spheres, respectively. The corresponding minimum values of Reynolds number \( N_R = \frac{\bar{u}_S d}{\nu} \) are 1600 and 2500 for the cubes and spheres, respectively. Thus the resistance of the porous media is properly expressed by equations 29 and 30 after substituting \( n = 0.21 \) and 0.23 and \( c = 50 \) and 62 for cubes and spheres, respectively, and \( N = 0.88 \).

27. Plots of \( \left( \frac{a_0}{a_1} \right)^{1-n} \) versus \( \left( \frac{L}{\lambda} \right) \left( \frac{a_0}{H} \right)^{1-n} \) for the test data are given in plates 15 through 18. The data have been fitted with straight lines, the slopes of the lines giving the measured values of \( \beta \). Table 4 gives a comparison between the values of \( \beta \) computed according to equation 30 and the measured values. The agreement is poor. Since the waves used for testing can hardly be classified as long waves, and since the study of transmission of long waves is beyond the scope of this study, no further comments on the discrepancies between the measured and calculated values of \( c \) and \( \beta \) will be presented.

Transmission of surface waves in porous media

28. In plates 4 through 9 the transmission coefficients obtained for cube and sphere assemblies of various lengths are plotted against wave height ratio \( 2a_0/H \) of the waves entering the porous structure. Smooth curves have been drawn through the data points, and focusing attention on \( 2a_0/H \) values of 0, 0.04, 0.10, and 0.20, the corresponding transmission coefficients for various \( L/\lambda \) values were read. These are presented in table 2 and constitute the proper material for the analysis. The
semilogarithmic representation of the transmission coefficients shown in
Table 2 is given in plates 10 through 14 in which $a_1/a_0$ is plotted
against $L/\lambda$ for the various wave height ratios $2a_0/H$ of 0, 0.04,
0.10, and 0.20. The alignment of the points for waves of infinitesimal
height is linear, and the transmission coefficient is

$$K_t = \frac{a_1}{a_0} = e^{-\frac{2a_0}{\lambda}} \quad (26 \, \text{bis and 44 \, bis)}$$

where the different values of $\alpha$ are given in Table 3.

29. If next it is assumed that the manner of loss is the same as
when the waves are long translation waves, from equation 25

$$\alpha = \frac{c\sqrt{T}}{2d^2 \left( \frac{\varepsilon}{1 - \varepsilon} \right)} \quad (25 \, \text{bis})$$

The value of $c$ can be calculated for different assemblies and the result
is given in Table 3. Again the agreement between the computed values of
$c$ given in Table 3 and the average value obtained from permeability tests,
c = 630 , is poor and shows that the manner of loss is different from that
for long translation waves. Consequently equation 25 is not valid for
surface waves.

30. The inferiorly placed curves in plates 10 through 14 relate to
waves of finite amplitude. Divergence from the upper limiting curves in¬
creases with increasing magnitude of the initial wave height $a_0$ . As
regards the waves of finite amplitude, equation 33 can be employed

$$\left( \frac{a_0}{a_1} \right)^{1-n} = 1 + \frac{1 - n}{2} cK \frac{v}{d_1} \left( \frac{gH}{\lambda} \right)^{1/n} \left( \frac{\varepsilon}{1 - \varepsilon} \right)^{1-n} \left( \frac{T}{\lambda} \right)^{1-n} \left( \frac{a_0}{H} \right)^{1-n} \frac{L}{\lambda} \quad (33 \, \text{bis})$$

which may be written as

$$\left( \frac{a_0}{a_1} \right)^{1-n} = 1 + \gamma \left( \frac{a_0}{H} \right)^{1-n} \frac{L}{\lambda} \quad (49)$$

21
\[ \gamma = \frac{1 - n}{2} cK \frac{\nu^n}{(\frac{e}{1 - e})^n l^{n+1}} (gH)^{1-n} \left(\frac{T}{\Lambda}\right)^{1-n} T \]  

(50)

31. As stated in paragraph 26, the minimum value of \( \bar{u}_s \) is 0.48 and 0.515 fps for cube and sphere assemblies, respectively. The corresponding minimum values of \( N_R \) are 1600 and 2500 for cube and sphere assemblies, respectively. Thus, the resistance of the porous media is properly expressed by equations 49 and 50 after substituting \( n = 0.21 \) and 0.23 and \( c = 50 \) and 62 for cube and sphere assemblies, respectively.

32. Plots of \( \left(\frac{a_0}{a_1}\right)^{1-n} \) versus \( \left(\frac{a_0}{H}\right)^{1-n}(L/\lambda) \) from the test data are given in plates 15 through 18. Fitting the data with straight lines gave the measured values of \( \gamma \). Substituting the \( \gamma \) values in equation 50 yielded the corresponding \( K \) values given in table 5. For most tests the value of \( K \) is fairly consistent with an average value of 0.595 and 0.273 for the cube and sphere assemblies, respectively.

### Wave Reflection Tests

33. The results of tests on reflection coefficients are highly scattered and do not follow a definite trend. For example, when the reflection coefficients were plotted against the wave heights (plates 19 and 20), the reflection coefficient showed a slight increase with increasing wave height in some tests and a slight decrease in others. The coefficient is apparently independent of the relative length of the porous structure and of the particle size. The independence of the reflection coefficient with regard to length of the porous structure is to be expected since the phenomenon of reflection is not affected by the thickness of the permeable structure. The reflection coefficient is independent of the thickness of the permeable structure as long as this thickness is larger than a few diameters of the grains constituting the porous medium. The independence of the reflection coefficient with regard to the grain size is rather paradoxical since in the case of turbulent flow, as was the case during
the tests reported herein, the amount of energy dissipated increases as the particle diameter increases (i.e. the larger the grain diameter, the greater the wave absorption by the porous structure). Small grains limit the motion within the porous media, causing less energy to be dissipated and more energy to be reflected.
34. Resistance to free surface steady flow in a porous medium such as a rubble-mound breakwater can be expressed as

$$\Delta E = k \frac{\rho u^3}{d}$$  (10 bis)

where $\rho$ is the density of the fluid, $u$ the approach velocity of the fluid, and $d$ a characteristic dimension (diameter) of the grains comprising the porous medium. The resistance coefficient $k$ is given by

$$k = f(N_R), \quad N_R = \frac{ud}{\nu}$$

where $\nu$ is the kinematic viscosity of the liquid. The possibility of determining the coefficient $k$ from the known law of resistance of a sphere was not explored since the purpose of this initial study was to determine the laws of flow through rubble-mound breakwaters where the shape of the rock may differ greatly from that of a sphere.

35. Permeability tests showed that for $N_R < 10$,

$$k = c\left(\frac{ud}{\nu}\right)^{-1}, \quad c = 630 \text{ for } \varepsilon = 40 \text{ percent} \quad (45 \text{ bis})$$

where $c$ is a function of the porosity $\varepsilon$ of the medium. Also, for $10 < N_R < 100$,

$$k = c\left(\frac{ud}{\nu}\right)^{-1/2}, \quad c = 170 \text{ for } \varepsilon = 40 \text{ percent} \quad (46 \text{ bis})$$

and for $N_R > 100$,

$$k = c\left(\frac{ud}{\nu}\right)^{-n} \quad (47 \text{ bis and 48 bis})$$

where $n = 0.21$ and $0.23$ and $c = 50$ and $62$ for cubic and spherical grains, respectively, the porosity being $40 \text{ percent}$. The values of $n$ and $c$ vary with the ratio between the length of the structure and the grain
diameter \((L/d)\). It was found that \(n\) and \(c\) increase for increasing values of \(L/d\).

36. The transmission coefficient for surface waves in a porous structure, \(K_t = a_1/a_0\), can be expressed for \(N_R > 100\) by

\[
\left( \frac{a_0}{a_1} \right)^{1-n} = 1 + \gamma \left( \frac{a_0}{H} \right)^{1-n} \frac{L}{\lambda} \tag{49\ bis}
\]

where

\[
\gamma = \frac{1 - n}{2} cK \frac{\nu^n}{\left( \frac{\varepsilon}{1 - \varepsilon} \right) d^n (gH)^{1-n} \left( \frac{T}{\lambda} \right)^{1-n} T} \tag{50\ bis}
\]

where \(n = 0.21\) and \(0.23\) and \(c = 50\) and \(62\) for cubic and spherical grains, respectively, the porosity being \(40\) percent. An average value of \(K\) is \(0.43\).

37. Future study should include:

a. Permeability tests to determine the effect of \(L/d\) on the values of \(n\) and \(c\). In these tests one grain size would be selected and the structure would be tested using different lengths. The upstream and downstream faces of the structure would be vertical. In these tests rock would be used as the porous material and the porosity would be maintained at \(40\) percent. In these permeability tests emphasis must be placed on small discharges \((N_R < 10)\) in order to define the values of \(n\) and \(c\) in the laminar and perhaps the transitional zone.

b. Transmission and reflection tests for translation and surface waves with infinitesimal as well as finite heights. These tests would be conducted using rock as the porous medium with a porosity of \(40\) percent. Again, the upstream and downstream faces of the structure would be kept vertical.

c. Tests similar to those mentioned in b above except using different slopes for the upstream and downstream faces of the structure.

38. Upon completion of the tests in a and b above, the different values obtained for \(n\) and \(c\) could be used for the scaling of the transmission and reflection coefficients.


5. Herbich, J. B., "Experimental Studies of Wave Filters and Absorbers," Project Report No. 44, Jan 1956, University of Minnesota, St. Anthony Falls Hydraulic Laboratory, St. Paul, Minn.


10. , "Wave Absorbers in Harbors," Contract Report No. 2-122, June 1965, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.


### Table 1

Values of c and n for Different Structures Tested

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Materials Used</th>
<th>d</th>
<th>L, ft</th>
<th>L/d</th>
<th>c</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-9</td>
<td>Concrete cubes</td>
<td>1-5/16</td>
<td>0.109</td>
<td>0.50</td>
<td>4.59</td>
<td>30</td>
</tr>
<tr>
<td>10-16</td>
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<td>1-5/16</td>
<td>0.109</td>
<td>0.90</td>
<td>8.26</td>
<td>20</td>
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<td>17-22</td>
<td></td>
<td>5/8</td>
<td>0.052</td>
<td>1.00</td>
<td>19.3</td>
<td>100</td>
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<tr>
<td>23-27</td>
<td></td>
<td>5/8</td>
<td>0.052</td>
<td>0.50</td>
<td>9.62</td>
<td>50</td>
</tr>
<tr>
<td>50-56</td>
<td>Rubber spheres</td>
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<td>1.00</td>
<td>4.81</td>
<td>40</td>
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<td>58-63</td>
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<td>2-1/2</td>
<td>0.208</td>
<td>2.00</td>
<td>9.62</td>
<td>65</td>
</tr>
<tr>
<td>64-69</td>
<td></td>
<td>2-1/2</td>
<td>0.208</td>
<td>3.26</td>
<td>15.67</td>
<td>80</td>
</tr>
<tr>
<td>70-78</td>
<td>Glass spheres</td>
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<tr>
<td>79-84</td>
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<td>0.54</td>
<td>10.36</td>
<td>80</td>
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<tr>
<td>105-110</td>
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<td>0.84</td>
<td>16.12</td>
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Table 2

Wave Transmission Coefficients

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<th>$2a/H$</th>
<th>$T$, sec</th>
<th>$K_t$</th>
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<tr>
<td></td>
<td></td>
<td>Rubber Spheres; $d = 2 - 1/2$ in.</td>
</tr>
<tr>
<td>0.00</td>
<td>0.71</td>
<td>0.46</td>
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<td>1.00</td>
<td>0.67</td>
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<tr>
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<td>1.41</td>
<td>0.75</td>
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<td>0.20</td>
<td>1.94</td>
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(Continued)
<table>
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<th>(2a_0/H)</th>
<th>(T, \text{ sec})</th>
<th>(K_t)</th>
<th>(L/\lambda = 0.112)</th>
<th>(L/\lambda = 0.199)</th>
<th>(L/\lambda = 0.110)</th>
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### Table 5

**K Values**

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<th>T, sec</th>
<th>Measured</th>
<th>Computed</th>
<th>K</th>
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LEGEND

○ SPHERE, GLASS
● SPHERE, RUBBER
□ CUBE, CONCRETE
△ LEAD SPHERE, BAKHMETEFF AND FEDOROFF

RESISTANCE COEFFICIENT (k)
VERSUS REYNOLDS NUMBER (N_R)

GLASS, RUBBER, AND LEAD SPHERES
AND CONCRETE CUBES
RESISTANCE COEFFICIENT ($k$) VERSUS REYNOLDS NUMBER ($N_R$)

GLASS AND RUBBER SPHERES
AND CONCRETE CUBES

PLATE 2
LEGEND
○ SPHERES
□ CUBES

n AND c VERSUS L/D

PLATE 3
TRANSMISSION COEFFICIENT (K_t) VERSUS 2a_o/H

RUBBER SPHERES; d = 2-1/2 IN.

NOTE: H = 1.0'
E = 40%
L = 1.0', 2.0', 3.26'
TRANSMISSION COEFFICIENT ($K_t$)
VERSUS $2a_o/H$
GLASS SPHERES; $d=1$ IN.
TRANSmission coefficient ($K_t$) versus $2a_o/H$

GLASS SPHERES; $d=5/8$ IN.

NOTE: $H = 1.0'$
$E = 40\%$
$L = 0.27'; 0.54'; 0.84'$

PLATE 6
NOTE: $H = 1.0'$
$\epsilon = 40\%$
$L = 0.50', 0.90'$

TRANSMISSION COEFFICIENT $(K_t)$
VERSUS $2a_o/H$

CONCRETE CUBES; $d = 1-5/16$ IN.
NOTE:  
$H = 1.0'$  
$\epsilon = 40\%$  
$L = 0.5', 1.0'$

TRANSMISSION COEFFICIENT ($K_t$)  
VERSUS $2\alpha_0/H$  
CONCRETE CUBES; $d=5/8$ IN.
NOTE: $H = 1.0'$
$\varepsilon = 40\%$
$L = 0.5', 1.0'$

TRANSMISSION COEFFICIENT ($K_t$) VERSUS $2a_o/H$
PLASTIC CUBES; $d=1/2$ IN.

PLATE 9
LEGEND

2a_o/H

• 0.00

○ 0.04

△ 0.10

□ 0.20

NOTE: H = 1.0'
ε = 40%
L = 1.0', 2.0', 3.26'

EXPONENTIAL LAW OF POROUS MEDIA DAMPING
RUBBER SPHERES; d=2-1/2 IN.

PLATE 10
T = 0.71 SEC

\[ \alpha = 2.50 \]

\begin{align*}
K_t & \approx 1.0 \\
L/\lambda & = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6
\end{align*}

T = 1.00 SEC

\[ \alpha = 2.86 \]

\begin{align*}
K_t & \approx 1.0 \\
L/\lambda & = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6
\end{align*}

T = 1.41 SEC

\[ \alpha = 3.38 \]

\begin{align*}
K_t & \approx 1.0 \\
L/\lambda & = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6
\end{align*}

T = 1.94 SEC

\[ \alpha = 4.38 \]

\begin{align*}
K_t & \approx 1.0 \\
L/\lambda & = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6
\end{align*}

LEGEND

\begin{align*}
2a_d/H & \\
\bullet & 0.00 \\
\circ & 0.04 \\
\triangle & 0.10 \\
\square & 0.20
\end{align*}

NOTE: H = 1.0 in.
E = 40%
L = 0.42', 0.80', 1.33'

EXPONENTIAL LAW OF POROUS MEDIA DAMPING
GLASS SPHERES; d = 1 in.

PLATE II
EXPONENTIAL LAW OF POROUS MEDIA DAMPING

GLASS SPHERES  d=5/8 IN.

NOTE: H = 1.0'
ε = 40%
L = 0.27; 0.54; 0.84'

LEGEND

\( \frac{2\alpha_0}{H} \)
- \( \bullet \) \( 0.00 \)
- \( \circ \) \( 0.04 \)
- \( \Delta \) \( 0.10 \)
- \( \square \) \( 0.20 \)
EXPONENTIAL LAW OF POROUS MEDIA DAMPING

CONCRETE CUBES; d = 1-5/16 AND 5/8 IN.
NOTE: $H = 1.0'$
$\varepsilon = 40\%$
$L = 0.5', 1.0'$

EXPONENTIAL LAW OF POROUS MEDIA DAMPING
PLASTIC CUBES; $d = 1/2$ IN.
NOTE: $H = 1.0'$
$\epsilon = 40\%$

POWER LAW OF POROUS MEDIA DAMPING
RUBBER SPHERES; $d = 2-1/2$ IN.
NOTE: $\epsilon = 40\%$
$H = 1.0$

POWER LAW OF POROUS MEDIA DAMPING
GLASS SPHERES; $d=5/8$ AND $1$ IN.
NOTE: H = 1.0', E = 40%

POWER LAW OF POROUS MEDIA DAMPING
PLASTIC CUBES; d=1/2 IN.
REFLECTION COEFFICIENT ($K_r$) VERSUS $2a_o$
GLASS AND RUBBER SPHERES
d=5/8, 1, AND 2-1/2 IN.
CONCRETE CUBES; \( d = \frac{5}{8} \) IN.

CONCRETE CUBES; \( d = 1\frac{5}{16} \) IN.

PLASTIC CUBES; \( d = \frac{1}{2} \) IN.

REFLECTION COEFFICIENT \( (K_r) \) VERSUS \( 2a_0 \)

CONCRETE AND PLASTIC CUBES
\( d = \frac{5}{8}, 1\frac{5}{16}, \) AND \( \frac{1}{2} \) IN.

PLATE 20
An expression is given for the resistance of free surface flow in a porous medium, such as a rubble-mound breakwater, in terms of the density of the fluid, the approach velocity, the grain diameter of the material in the breakwater structure, and a resistance coefficient which is a function of Reynolds number of the flow. Tests indicated that the length of the structure relative to the grain diameter is an important factor in determining the resistance of free surface flow in a porous medium. A certain ratio between the length of the structure and its grain diameter must be reached before the structure can be treated as a porous medium. An expression derived by Dr. G. H. Keulegan for damping of translation waves by screen filters was modified to give the transmission coefficient of surface waves propagated through a porous structure. The constants in this expression were also found to be a function of the length of the structure relative to the grain diameter. Measured and computed values of wave transmission coefficients agreed well.
<table>
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<th>LINK C</th>
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