Creep of Ice at Low Stresses
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by T. R. Butkovich and J. K. Landauer
PREFACE

This is one of a series of reports on USA SIPRE task 022.01.039 (formerly 022.01.008), Mechanics of deformation of snow and ice. The purpose of these investigations is to formulate and express, in a manner suitable for general engineering purposes, the deformation of snow and ice masses and closure of cavities with time under stress.

The experiments were performed, and the report prepared, by Mr. Butkovich and Dr. Landauer. Work on this project was performed for USA SIPRE'S Basic Research Branch, Mr. J. A. Bender, chief.

This report has been reviewed and approved for publication by the Office of the Chief of Engineers.

W. L. NUNDESSER
Colonel, Corps of Engineers
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SUMMARY

Low stress creep measurements were made on commercial ice and small-grained glacier ice in the temperature range from -1.3 to -18.9°C at stresses down to about $1 \times 10^4$ dynes/cm$^2$. The uniaxial stresses and strain rates were reduced to shear stresses and shear strain rates by respectively dividing and multiplying the former by $\sqrt{3}$.

The log shear strain rate vs log shear stress curves was essentially linear for the low-stress creep measurements. Assuming a linear flow law for low-stress creep, activation energies for creep of about 14,300 cal/mole were determined. It can also be seen that the smaller-grained ice has a higher viscosity coefficient.

The observed activation energy for creep of ice is probably that for self-diffusion. Although lacking a concrete deformation mechanism, the rate process theory, which leads to a hyperbolic sine stress dependence, seems to best describe the experimental results.
Most engineering materials are assumed to have a definite yield stress below which no plastic deformation will occur. Usually a careful study will show that creep occurs at stresses below the yield stress. The lowest stresses for which laboratory creep experiments have been made on ice are about $2 \times 10^5$ dynes/cm$^2$ (Jellinek and Brill, 1956; Steinemann, 1954). Creep at still lower stresses has been measured in glaciers. Gerrard, Perutz, and Roch (1952) report inclinometer measurements at shear stresses of about $1 \times 10^5$ dynes/cm$^2$.

Butkovich and Landauer (1958) have made measurements of the minimum creep rate as a function of stress in the range of 0.5 - 15 kg/cm$^2$ on similar types of ice to that used for this investigation. From these results an empirical flow law for ice was derived. The present work was undertaken to measure creep rates at stresses below those previously used, and perhaps shed some light on a mechanism to explain steady state creep of ice.

Electrical strain gages of the type used in these experiments make it possible to measure extremely small displacements, and they also have the required stability to measure these displacements over long periods of time. An apparatus using these gages was designed to measure creep at shear stresses down to the order of $1 \times 10^4$ dynes/cm$^2$. Stress and temperature dependence of the low stress creep rates was also investigated.

**EXPERIMENTAL**

An apparatus was designed so that three specimens could be measured simultaneously at different stresses. The ice samples were cut to approximately $2 \times 2 \times 6$ cm and were compressed uniaxially by weights hanging from the end of an aluminum beam (Fig. 1). A steel ball bearing at the center of the beam transmitted the force through an aluminum cap frozen to the specimen. Electrical strain gages (Statham Models G-7 and G-7A) were mounted so as to measure the deflection of the end of the
beam. The gage and beam supports were mounted on a heavy brass plate which was placed on shock pads to reduce vibration. The entire apparatus was inclosed in an insulated cabinet. Temperature control was provided by thermostat regulation of electrical heating and by a cold brine radiator. Two small fans prevented stratification of the air within the cabinet.

For most of the tests, the individual ice specimens were covered with thin flexible latex sleeves to minimize sublimation. In addition to this, ice chips were liberally spread throughout the cabinet.

Temperature was measured with a mercury-in-glass thermometer, calibrated in 0.1°C, which could easily be read to 0.05°C. Because of the large thermal expansion coefficient of ice, a change in temperature was easily detected on the strain gages. For each run, temperatures were held within ±0.5°C. Strain readings were always corrected to a standard temperature, usually the mid-point between the maximum and minimum temperature within a given run. The temperature corrections for 0.05°C amounted to about 6 μin/in, which is about the limit of accuracy in the strain gage readings made with a Baldwin Type M Strain Indicator.

Four Stathan gages were used in the deformation measurements. A new gage replaced one that became defective. These gages were calibrated at -20°C with a drum micrometer which could be read to 0.001 cm. These calibrations are independent of temperature over a large range.

The applied shear stress was determined from the applied load on the ice specimen. The applied load included the mass of the beam, ball bearing, the top cap on the ice cylinder, and one-half the weight of the ice prism, plus twice the sum of the applied weight and the force due to the strain gage. As the ice deforms half as much as the end of the beam moves, the deformation was obtained by taking half of that indicated and multiplying by the calibration factor for the corresponding strain gage.

Three different groups of ice specimens were used. The first group, designated as MP1, was small-grained, randomly oriented ice, average grain size of 3 mm diam, with more or less homogeneously distributed, irregularly shaped air bubbles, and with an average density of 0.905 g/cm³ (Butkovich, 1959). The other two groups were commercial, artificial ice and were designated C-1 and C-2. This ice is comparatively large-grained, bubble-free with one axis elongated, with grains averaging 1 - 2 cm diam and 4 - 5 cm long and with a density of 0.917 g/cm³. The orientation of C-1 ice is unknown; however, C-2 ice had the elongated axis parallel to the direction of application of the load with the c-axes predominately normal to this direction.

During the first group of tests, several modifications were made on the original apparatus to increase the stability of the gages. In the early runs, the gages tended to drift, and sometimes indicated large deformation readings. Probably this erratic behavior was due to mechanical instabilities in the apparatus. Only those tests of the first group that appeared to give reasonable results are shown. Figure 2 is a typical deformation-time curve. The straight line portion shown corresponds to the minimum creep rate.

![Figure 2. Typical deformation-time curve. C-1 ice; temp -6.1°C.](image-url)
After a short initial period, the deformation-time curves were essentially linear for all cases reported at these low stresses. The uniaxial strain rates were determined from the slopes of the lines. The shear stress $\tau$ was obtained by dividing the uniaxial stress $\sigma$ by $\sqrt{3}$, and the shear strain rate $\dot{\gamma}$ was obtained by multiplying the uniaxial strain rate by $\sqrt{3}$ (Nye, 1953). Table I shows the results of the tests on the three groups of ice between -1.3 and -18.9°C.

Figure 3 shows the plot of log $\tau$ vs log $\dot{\gamma}$ for each run. Each curve could be best represented by straight lines with slopes varying between 0.86 and 1.15. This plot also shows an extrapolation to low stresses of the higher stress experiments of Butkovich and Landauer (1958) at -5°C on similar ices. In the above reference, it was shown that a power law stress dependence gave the best fit to the data, although cubic plus linear and hyperbolic sine equations were also fitted. In Figure 3, the averaged cubic plus linear and averaged hyperbolic sine constants for polycrystalline ices represent the low-stress experiments very well, while the power law does not.

Assuming a linear low-stress flow law (average slope of log $\tau$ vs log $\dot{\gamma}$ equals 1), the viscosity has been calculated and is plotted in Figure 4. From this, an activation energy for creep of 14,300 cal/mole was obtained. This agrees with most of the previously reported results for snow and ice (Jellinek and Brill, 1956; Rarety and Tabor, 1958; Landauer, 1955; and Landauer, 1957). In addition, this plot indicates that the smaller-grained (MPI) ice has a higher viscosity coefficient than the larger-grained (C-1 and C-2) ice, as is to be expected.

The observed activation energy for creep of ice is probably the activation energy for self-diffusion. The self-diffusion energy can be obtained approximately from the melting temperature. According to Dorn (1956), this should be about 11,000 cal/mole for ice.
<table>
<thead>
<tr>
<th>Temp (C)</th>
<th>Date run began</th>
<th>Type of ice</th>
<th>Time of run (hr)</th>
<th>Specimen length (cm)</th>
<th>Uniaxial stress, ( \sigma ) (dynes/cm(^2))</th>
<th>Minimum uniaxial strain rate, ( \dot{\varepsilon} ) (sec(^{-1}))</th>
<th>Shear stress, ( \tau ) (dynes/cm(^2))</th>
<th>Minimum shear strain rate, ( \dot{\gamma} ) (sec(^{-1}))</th>
<th>Viscosity, ( \eta ) (poises)</th>
<th>Strain gage no.</th>
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<td>-1.3</td>
<td>3/59</td>
<td>C-2</td>
<td>217</td>
<td>6.0</td>
<td>208 x 10(^3)</td>
<td>9.320 x 10(^{-10})</td>
<td>120 x 10(^3)</td>
<td>16.142 x 10(^{-10})</td>
<td>0.743 x 10(^{14})</td>
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<tr>
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<td>168</td>
<td>6.0</td>
<td>208</td>
<td>7.336</td>
<td>120</td>
<td>12.706</td>
<td>0.642</td>
<td>744</td>
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<tr>
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<td>8/58</td>
<td>C-1</td>
<td>1004</td>
<td>5.8</td>
<td>208</td>
<td>4.78</td>
<td>120</td>
<td>8.28</td>
<td>0.944</td>
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<td>6/58</td>
<td>MPl</td>
<td>626</td>
<td>5.7</td>
<td>207</td>
<td>0.949</td>
<td>120</td>
<td>9.44</td>
<td>7.30</td>
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<td>-11.0</td>
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</table>
Most of the high temperature creep theories based on dislocation models (Mott, 1956; Weertman, 1957) give a creep rate depending on stress to a power of 3 or 4. Theories based on rate processes (Kauzmann, 1941) or diffusion of vacancies (Nabarro, 1947; Herring, 1950) predict linear behavior at low stress. Although lacking a concrete deformation mechanism, the rate process theory, which leads to a hyperbolic sine stress dependence, seems to best describe the experimental results.
REFERENCES


